Int. J. Multiphase Flow Vol. 19, No. 2, pp. 369–376, 1993
Printed in Great Britain. All rights reserved

0301-9322/93 \$6.00 + 0.00

BRIEF COMMUNICATION

DIMENSIONLESS FORM OF A ONE-DIMENSIONAL WAVE MODEL FOR THE STRATIFIED FLOW REGIME TRANSITION

C. J. CROWLEY, G. B. WALLIS and J. J. BARRY
Creare Inc., P.O. Box 71, Hanover, NH 03755, U.S.A.

(Received 14 April 1992; in revised form 30 November 1992)

INTRODUCTION

Crowley *et al.* (1992) present a complete solution for a one-dimensional wave theory model for the transition from stratified to slug or annular flow regimes—a complete set of equations and a solution methodology. This discussion follows up the previous work by presenting this one-dimensional wave theory and results in a dimensionless form. Why is the result in dimensionless form useful? Figure 1 illustrates a typical design map of the transition in dimensionless form generated for:

- A fixed pipe inclination (horizontal in this case).
- Constant wall–liquid and wall–gas friction factors ($f_{WL} = f_{WG} = 0.005$).
- Turbulent flow of both phases.

For these conditions, the dimensionless variables derived in the analysis are:

- Liquid phase Froude number, j_L^*.
- Gas phase Froude number, j_G^*.
- Liquid–gas density ratio, $R = \rho_L/\rho_G$.
- Interfacial friction factor ratio, f_i/f_{WG}.

Figure 1 shows that for $f_i/f_{WG} = 1$—equivalent to assuming a smooth interface between the gas and the liquid phases—the one-dimensional wave transition can be plotted on a dimensionless map of liquid phase Froude number vs gas phase Froude number, with the density ratio as a parametric variable. Similar maps could be generated for other pipe inclinations (as shown later) to create a set of universal flow regime maps in dimensionless coordinates.

Figure 1 illustrates the two main reasons why the one-dimensional wave approach is instructive:

- First, the dependence on the liquid–gas density ratio becomes apparent. As this density ratio decreases, i.e. as the gas density increases, the transition lies at a higher liquid Froude number. This effect of density ratio is most important at values of $j_G^* < 0.1$. The values of the density ratio in figure 1 cover the range from air and water at atmospheric pressure ($R = 900$) to values typical of gas and oil pipelines ($R = 30$).
- Second, the Taitel–Dukler (1976) model for the transition, represented by the dashed line in figure 1, is seen to be an approximate representation of the more general one-dimensional wave theory for the condition of low gas density ($R = 900$) only.

The mechanistic analysis proposed by Taitel & Dukler (1976) is widely used for the prediction of this flow regime transition. [Ferschneider *et al.* (1985) first pointed out that the Taitel–Dukler model is actually a special case of the more general one-dimensional wave analysis.] Taitel &

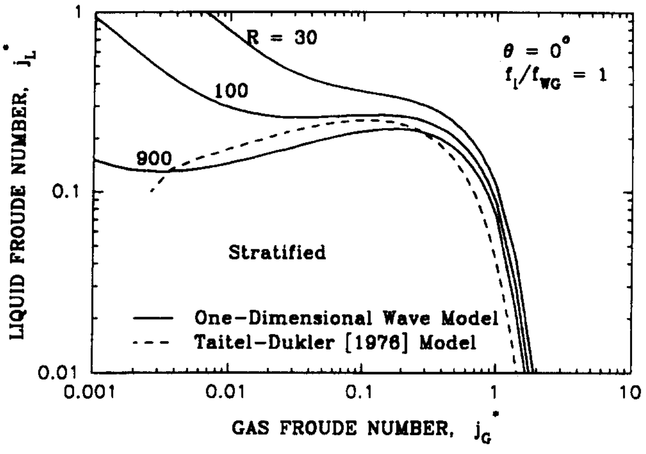

Figure 1. Dimensionless stratified regime transition for a horizontal pipe.

Dukler (1976) showed that the simplified transition model could be plotted on a dimensionless map in coordinates of the gas phase Froude number and the Martinelli parameter. Since the Martinelli parameter is a ratio of the gas and liquid phase Froude numbers if the friction factors are constant, figure 1 is equivalent to that result. Figure 1 clearly shows the correspondence between the one-dimensional wave model presented here and the Taitel–Dukler approximation. There is an additional dependence upon the density ratio in the one-dimensional model in these dimensionless coordinates.

The remainder of this discussion presents the equations in dimensionless form, illustrates the sensitivity of the calculations to the key modelling parameters and compares selected data with the model on dimensionless regime maps.

DEFINITION OF THE GENERAL DIMENSIONLESS VELOCITIES

First, define the following dimensionless velocities:

$$j_L^* = \frac{U_{LS}\rho_L^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}}, \qquad [1]$$

$$j_G^* = \frac{U_{GS}\rho_G^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}}, \qquad [2]$$

$$c^* = \frac{c\rho_L^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}}, \qquad [3]$$

$$V_0^* = \frac{V_0\rho_L^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}} \qquad [4]$$

and

$$V_w^* = \frac{V_w\rho_L^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}}, \qquad [5]$$

where V_0 and V_w are components which make up the dynamic wave velocity v_w, and c represents the continuity wave velocity. In essence, all of the velocities except U_{GS} are scaled the same and are defined as Froude numbers for the liquid phase. The parameter j_G^* in [2] can be put on the same basis by introducing the density ratio parameter

$$R = \frac{\rho_L}{\rho_G}. \qquad [6]$$

Then [2] becomes

$$j_G^* R^{0.5} = \frac{U_{GS}\rho_L^{0.5}}{[gD(\rho_L - \rho_G)]^{0.5}}. \qquad [7]$$

SUMMARY OF THE DIMENSIONLESS EQUATIONS

This section summarizes the equations of the solution in dimensionless form. Reference is made to the corresponding dimensional equations in the previous work (Crowley *et al.* 1992), e.g. [C6] refers to [6] in Crowley *et al.* (1992).

Equilibrium model for stratified flow

First, an equation representing the steady-state solution for the void fraction in a stratified flow is needed as a foundation for the analysis. The previous dimensional [C6] becomes, in dimensionless form,

$$F' = \left(\frac{\pi^2 f_{WL}\tilde{S}_L j_L^{*2}}{32\tilde{A}_L^3}\right) - \left(\frac{\pi^2 f_{WG}\tilde{S}_G j_G^{*2}}{32\tilde{A}_G^3}\right)$$
$$\mp \left(\frac{f_i}{f_{WG}}\right)\left(\frac{\pi^3 f_{WG}\tilde{S}_i}{128}\right)\left(\frac{j_G^{*2}}{\tilde{A}_L\tilde{A}_G^3} - \frac{2j_G^* j_L^*}{R^{0.5}\tilde{A}_L^2\tilde{A}_G^2} + \frac{j_L^{*2}}{R\tilde{A}_L^3\tilde{A}_G}\right) + \sin\theta, \qquad [8]$$

where $F' = F/[g(\rho_L - \rho_G)]$. All of the dimensionless geometric terms $\tilde{A}_L$, $\tilde{A}_G$, $\tilde{S}_L$, $\tilde{S}_G$ and $\tilde{S}_i$ in [8] are functions of only the dimensionless liquid level in the pipe h^*, defined previously as [C7]–[C11]. Equations [1] and [7] herein define the phase Froude numbers.

Solution for the liquid Froude number

The solution procedure requires that the liquid phase Froude number for a steady-state solution ($F' = 0$) be calculated, assuming that j_G^* has a given value and the dimensionless liquid level h^* in stratified flow is known. Assuming, in addition, that the friction factors f_{WG} and f_{WL}, the density ratio R and the interfacial friction factor ratio (f_i/f_{WG}) are constants, then the right-hand side of [8] can be set equal to zero and rewritten as a quadratic to solve for j_L^*:

$$a j_L^{*2} \pm 2b j_L^* + d = 0, \qquad [9]$$

where the coefficients in [9] are defined by

$$a = \left[\frac{4f_{WL}\tilde{S}_L}{\pi\tilde{A}_L^3} \mp \frac{f_i\tilde{S}_i}{R\tilde{A}_L^3\tilde{A}_G}\right), \qquad [10]$$

$$b = \pm\left(\frac{f_i\tilde{S}_i j_G^*}{R^{0.5}\tilde{A}_L^2\tilde{A}_G^2}\right) \qquad [11]$$

and

$$d = \left(\frac{128\sin\theta}{\pi^3}\right) - \left(\frac{4f_{WG}\tilde{S}_G j_G^{*2}}{\pi\tilde{A}_G^3}\right) \mp \left(\frac{f_i\tilde{S}_i j_G^{*2}}{\tilde{A}_L\tilde{A}_G^3}\right). \qquad [12]$$

Continuity wave velocity

In order to use the stability criterion developed for the one-dimensional wave model, we need expressions for the continuity and dynamic wave velocities (v_w and c) in dimensionless form. Equation [C15] for the continuity wave velocity becomes dimensionless as

$$v_w^* = (V_w^* - V_0^*), \qquad [13]$$

where the dimensionless reference velocity V_0^* (formerly [C16]) is given by

$$V_0^* = \frac{\left(\frac{\pi}{4}\right)\left(\frac{j_L^*}{\tilde{A}_L^2} + \frac{j_G^*}{\tilde{A}_G^2 R^{0.5}}\right)}{\left(\frac{1}{\tilde{A}_L} + \frac{1}{R\tilde{A}_G}\right)}. \qquad [14]$$

The general relationship used to derive the wave velocity V_w ([C17]) becomes, in dimensionless form,

$$V_w^* = -\left(\frac{\pi}{4}\right)\left(\frac{\mathrm{d}F\,\mathrm{d}A}{\mathrm{d}F\,\mathrm{d}L - \mathrm{d}F\,\mathrm{d}G}\right) \quad [15]$$

and the derivatives in [15] (formerly [C18]–[C20]) are now:

$$\mathrm{d}F\,\mathrm{d}L = \frac{\left(\frac{4}{\pi}\right)^2\left(\frac{\partial F}{\partial U_{LS}}\right)}{[g\rho_L(\rho_L-\rho_G)D]^{0.5}} = \left(\frac{f_{WL}\tilde{S}_L j_L^*}{\tilde{A}_L^3}\right) \pm\left(\frac{f_i}{f_{WG}}\right)\left(\frac{\pi f_{WG}\tilde{S}_i}{4\tilde{A}_L\tilde{A}_G}\right)\left(-\frac{j_L^*}{R\tilde{A}_L^2} + \frac{j_G^*}{R^{0.5}\tilde{A}_L\tilde{A}_G}\right), \quad [16]$$

$$\mathrm{d}F\,\mathrm{d}G = \frac{\left(\frac{4}{\pi}\right)^2\left(\frac{\partial F}{\partial U_{GS}}\right)}{[g\rho_L(\rho_L-\rho_G)D]^{0.5}} = \left(\frac{f_{WG}\tilde{S}_G j_G^*}{R^{0.5}\tilde{A}_L^3}\right) \mp\left(\frac{f_i}{f_{WG}}\right)\left(\frac{\pi f_{WG}\tilde{S}_i}{4\tilde{A}_L\tilde{A}_G}\right)\left(-\frac{j_L^*}{R\tilde{A}_L\tilde{A}_G} + \frac{j_G^*}{R^{0.5}\tilde{A}_G^2}\right) \quad [17]$$

and

$$\begin{aligned}\mathrm{d}F\,\mathrm{d}A &= \frac{\left(\frac{4}{\pi}\right)^2\left(\frac{\partial F}{\partial \tilde{A}_L}\right)}{g(\rho_L-\rho_G)} \\ &= \left(\frac{f_{WL}j_L^{*2}}{2}\right)\left[\left(\frac{\partial \tilde{S}_L}{\partial \tilde{A}_L}\right)\left(\frac{1}{\tilde{A}_L^3}\right) - \frac{3\tilde{S}_L}{\tilde{A}_L^4}\right] - \left(\frac{f_{WG}j_G^{*2}}{2}\right)\left[\left(\frac{\partial \tilde{S}_G}{\partial \tilde{A}_L}\right)\left(\frac{1}{\tilde{A}_G^3}\right) + \frac{3\tilde{S}_G}{\tilde{A}_G^4}\right] \\ &\mp\left(\frac{\pi f_{WG}}{8}\right)\left(\frac{f_i}{f_{WG}}\right)\left\{j_G^{*2}\left[\left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L}\right)\left(\frac{1}{\tilde{A}_L\tilde{A}_G^3}\right) - \frac{\tilde{S}_i}{\tilde{A}_L^2\tilde{A}_G^3} + \frac{3\tilde{S}_i}{\tilde{A}_L\tilde{A}_G^4}\right]\right. \\ &- \frac{2j_L^*j_G^*}{R^{0.5}}\left[\left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L}\right)\left(\frac{1}{\tilde{A}_L^2\tilde{A}_G^2}\right) - \frac{2\tilde{S}_i}{\tilde{A}_L^3\tilde{A}_G^2} + \frac{2\tilde{S}_i}{\tilde{A}_L^2\tilde{A}_G^3}\right] \\ &\left.+\frac{j_L^{*2}}{R}\left[\left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L}\right)\left(\frac{1}{\tilde{A}_L^3\tilde{A}_G}\right) - \frac{3\tilde{S}_i}{\tilde{A}_L^4\tilde{A}_G} + \frac{\tilde{S}_i}{\tilde{A}_L^3\tilde{A}_G^2}\right]\right\}.\end{aligned} \quad [18]$$

Note that these explicit derivatives in [16]–[18] are only valid if constant friction factors are assumed. Equation [18] requires additional derivatives of the geometric parameters defined previously as [C21]–[C23].

Dynamic wave velocity

The equation for the square of the dynamic wave velocity is obtained by taking the previous dimensional [C24] for c^2 and using the definition in [3] to get

$$c^{*2} = \left[\frac{\left(\frac{\cos\theta}{\tilde{S}_i}\right) - \frac{\left[\frac{\pi}{4}\right]^2\left[\frac{R^{0.5}j_G^*}{\tilde{A}_G} - \frac{j_L^*}{\tilde{A}_L}\right]^2}{(\tilde{A}_L + \tilde{A}_G R)}}{\left(\frac{1}{\tilde{A}_L} + \frac{1}{R\tilde{A}_G}\right)}\right]. \quad [19]$$

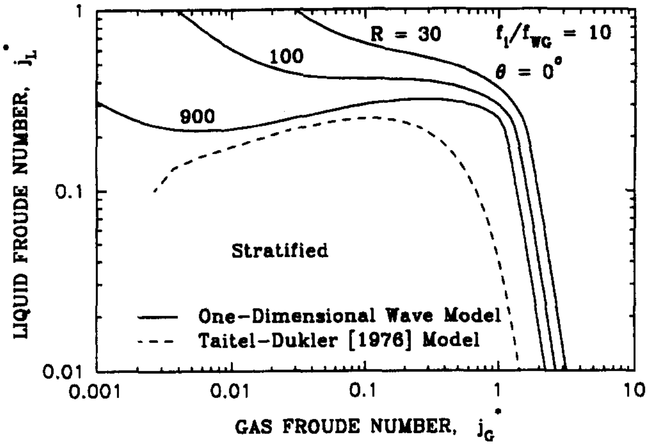

Figure 2. Effect of interfacial friction on the dimensionless stratified transition for a horizontal pipe.

SOLUTION PROCEDURE

Since dynamic waves move with a velocity of $\pm c^*$ relative to the weighted mean velocity V_0^*, while continuity waves move only in one direction at the velocity v_w^*, the criterion for instability involves the squares of the velocities:

$$v_w^{*2} > c^{*2}. \qquad [20]$$

The stratified-to-slug flow regime transition occurs when the two velocities are equal in [20]. The equations for v_w^* and c^* depend upon the liquid fraction and both superficial phase velocities, and it is not possible to solve either equation explicitly for one parameter in terms of the other two parameters. Therefore, an iterative solution procedure is required to find the flow regime transition. The solution procedure is the same as described in Crowley *et al.* (1992), except that the dimensionless forms of the equations as described in section 4 may be substituted for the dimensional versions.

TYPICAL MODEL SENSITIVITY AND DATA COMPARISONS

Figure 2 shows the sensitivity to the interfacial friction factor by assuming a high value of $f_i/f_{WG} = 10$ in the one-dimensional wave model. Comparing figures 1 and 2, the effect is quite small for horizontal pipes. In general, the transition values of the liquid Froude number vary no more than a factor of 2 at $j_G^* < 1.0$ for this range $1 < (f_i/f_{WG}) < 10$. Values of j_G^* at the transition vary by only a factor of 2 over this same range of interfacial friction factor for $j_G^* > 1$.

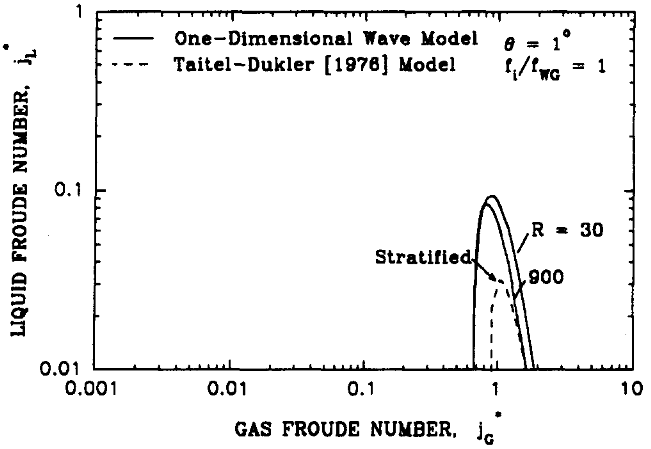

Figure 3. Dimensionless stratified regime transition for an upwardly inclined pipe.

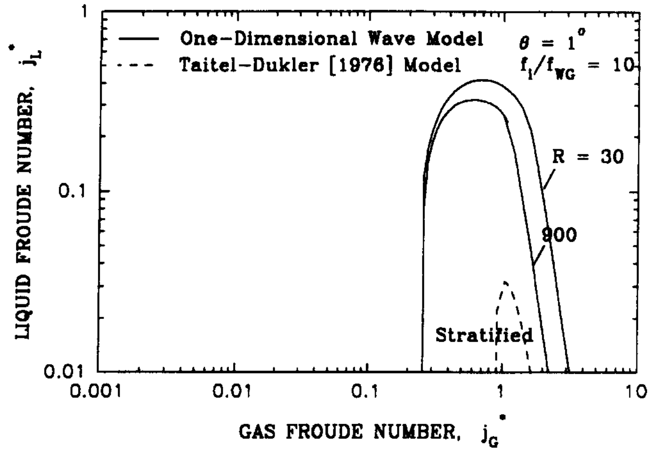

Figure 4. Effect of interfacial friction on the dimensionless stratified transition for an upwardly inclined pipe.

Figures 3 and 4 show similar dimensionless transition boundaries for a 1° upward inclination. Here, the interfacial friction factor ratio in the range of $1 < (f_i/f_{WG}) < 10$ is more significant. The peak value of the liquid Froude number at the transition boundary varies by about a factor of 5 over this range of the friction factor ratio.

Figures 3 and 4 also show that the transition boundary for the upwardly inclined pipes is not sensitive to the liquid–gas density ratio. This is because the solution does not exist in the same range of gas Froude number ($j_G^* < 0.1$), where the density ratio matters in the horizontal pipe (figures 1 and 2).

Crowley *et al.* (1992) presented extensive data comparisons showing that this trend of the one-dimensional wave theory is consistent with a wide range of experimental data. Data for both the effects of pipe inclination and a broad range of density ratios are available from the same test facility at large, 0.17 m, diameter (Crowley & Sam 1986). Therefore, we illustrate the comparisons of the dimensionless form of the one-dimensional wave model only with selected data from those experiments here.

Figure 5 shows that the transition at low gas density ($R = 620$) is predicted well in a horizontal pipe, and the result is close to the Taitel-Dukler model (dashed line). Figure 6 shows that the transition is also predicted well at gas densities typical of gas and oil pipelines ($R = 31$). The

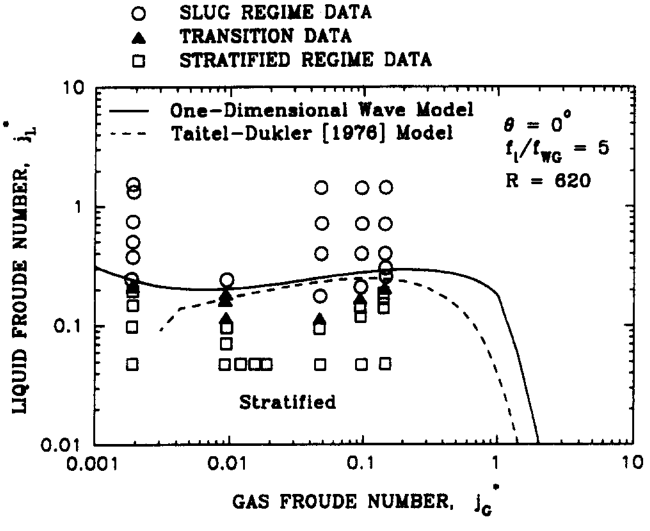

Figure 5. Dimensionless stratified transition for a horizontal 0.17 m diameter pipe at low gas density (Crowley & Sam 1986).

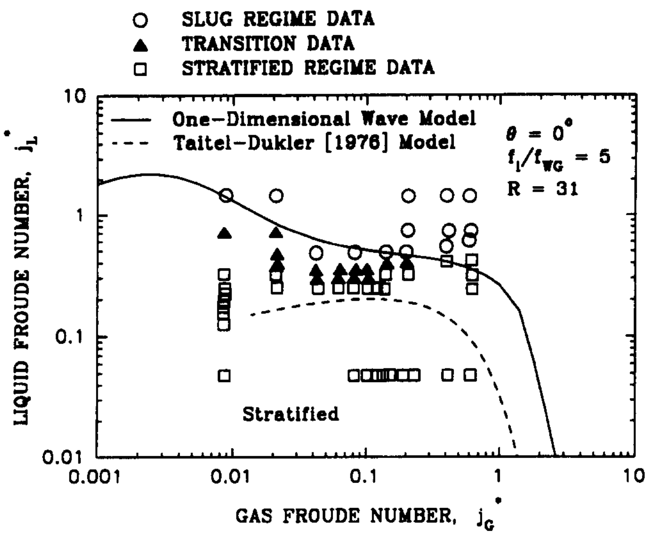

Figure 6. Dimensionless stratified transition for a horizontal 0.17 m diameter pipe at high gas density (Crowley & Sam 1986).

transition lies at higher liquid Froude numbers at a high gas density (figure 6) than at low gas density (figure 5). The larger liquid Froude numbers and the upward trend of the transition at low gas Froude number are predicted by the one-dimensional wave analysis, but not the Taitel–Dukler (1976) model. One previous approach to account for this effect of the gas density was to increase the interfacial shear assumed in the Taitel–Dukler model. This approach has the right effect to achieve comparisons with the data, but the correct physical result is contained in the one-dimensional wave theory, without modification of the interfacial friction. Therefore, the one-dimensional wave theory is a more general approach, better applicable to conditions of low gas velocity and high gas density.

The one-dimensional wave model also predicts the observed transition for upward pipe inclinations (figure 7) at 2°. Because of the large sensitivity of the model to the interfacial friction factor ratio for upwardly inclined pipes (figures 3 and 4), these data in figure 7 have been used to select the recommended value of $f_i/f_{WG} = 5$ in the analysis. That value is used for the one-dimensional wave model in figures 5 and 6.

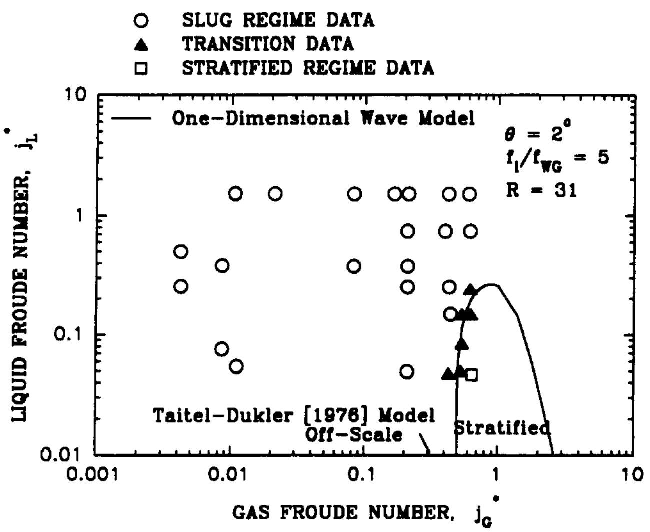

Figure 7. Dimensionless stratified transition for an upwardly inclined 0.17 m diameter pipe at high gas density (Crowley & Sam 1986).

REFERENCES

CROWLEY, C. J. & SAM, R. G. 1986 Investigation of two-phase flow in horizontal and inclined pipes at large pipe size and high gas density. Creare Inc. TN-399, Pipeline Research Committee Project PR 172-507, American Gas Association Catalog No. L51509.

CROWLEY, C. J., WALLIS, G. B., & BARRY, J. J. 1992 Validation of a one-dimensional wave model for the stratified-to-slug flow regime transition, with consequences for wave growth and slug frequency. *Int. J. Multiphase Flow* **18,** 249–271.

FERSCHNEIDER, G., LAGIERE, M., BOURGEOIS, T. & FITREMANN, J-M. 1985 How to calculate two-phase flow of gas and oil in pipe lines. *Pipeline Ind.* **63,** 33–39.

TAITEL, Y. & DUKLER, A. E. 1976 A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. *AIChE Jl* **22,** 47–56.

WALLIS, G. B., 1969 *One-dimensional Two-phase Flow.* McGraw-Hill, New York.

WU, H. L., POTS, B. F. M., HOLLENBERG, J. F. & MEERHOFF, R. 1987 Flow pattern transitions in a two-phase gas/condensate flow at high pressure in an 8-inch horizontal pipe. In *Proc. 3rd Int. Conf. on Multi-phase Flow*, The Hague, The Netherlands, pp. 13–21.